積分法（数学II）
問題編Part2

◦ 定積分の利用

問1　関数 $f(x)$ が次の関係を満たすとき，$f(x)$ を求めよ.

(1) $f(x) = 3x^2 - 2x + \underline{\int_{-1}^{1} f(t)\,dt}$

Point 関数の決定問題では $\int_{定数}^{定数} f(t)\,dt$ を文字(定数)でおく

$\int_{-1}^{1} f(t)\,dt = A$

$f(x) = 3x^2 - 2x + A$

$A = \int_{-1}^{1} (3t^2 - 2t + A)\,dt$

$= \left[t^3 - t^2 + At \right]_{-1}^{1}$

$= 1 - 1 + A - (-1 - 1 - A)$

$= 2A + 2$

$A = 2A + 2$

$A = -2$

$f(x) = 3x^2 - 2x - 2$

(2) $f(x) = 3x + \underline{\int_0^1 (x+t)f(t)\,dt}$

$\rightarrow \int_0^1 (x+t)f(t)\,dt$

$= x\underbrace{\boxed{\int_0^1 f(t)\,dt}}_{=A} + \underbrace{\boxed{\int_0^1 t f(t)\,dt}}_{=B}$

$f(x) = 3x + Ax + B$

$= (3+A)x + B$

$A = \int_0^1 \{(3+A)t + B\}\,dt$

$= \left[\left(\frac{3+A}{2}\right)t^2 + Bt\right]_0^1$

$= \frac{3+A}{2} + B$

$A - 2B = 3$ — ①

$\rightarrow B = \int_0^1 t\{(3+A)t + B\}\,dt$

$= \int_0^1 \{(3+A)t^2 + Bt\}\,dt$

$= \left[\frac{3+A}{3}t^3 + \frac{B}{2}t^2\right]_0^1$

$= \frac{3+A}{3} + \frac{B}{2}$

$2A - 3B = -6$ — ②

①, ②より

$A - 2B = 3$

$2A - 3B = -6$

$A = -21,\ B = -12$

$f(x) = (3-21)x - 12$

$= \underline{-18x - 12}$ //

問2　関数 $f(x)$ が $\int_a^x f(t)\,dt = x^3 + x^2 + x + 1$ を満たすとき、①$f(1)$の値を求めよ。また、②実数の定数 a の値を求めよ。

$$\left\{\int_a^x f(t)\,dt\right\}' = \{F(x) - F(a)\}'$$

$$= F'(x)$$

$$= f(x)$$

$$\frac{d}{dx}\int_a^x f(t)\,dt = \frac{d}{dx}(x^3+x^2+x+1)$$

$$= 3x^2+2x+1 = f(x)$$

$$f(1) = 3+2+1 = 6$$

$$\int_a^a f(t)\,dt = \underline{a^3+a^2+a+1}$$

$$\|$$

$$\underline{0}$$

$$a^3+a^2+a+1=0$$

$$\begin{array}{r|rrrr} -1 & 1 & 1 & 1 & 1 \\ & & -1 & 0 & -1 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

$$(a+1)(\underbrace{a^2+1}_{\neq 0}) = 0$$

$$a+1 = 0$$

$$\underline{a=-1}$$

Point

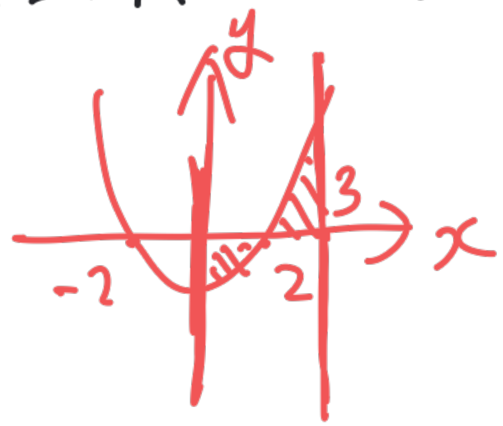
$$\boxed{\frac{d}{dx}\int_a^x f(t)\,dt = f(x)}$$

$$\rightarrow \frac{d}{dx}\int_x^a f(t)\,dt = -f(x)$$

問3 次の曲線や直線とx軸で囲まれた部分の面積を求めよ

面積

$y = x^2 - 4 \quad (x \geqq 0)$, $x=0$, $x=3$



1st 図を描く

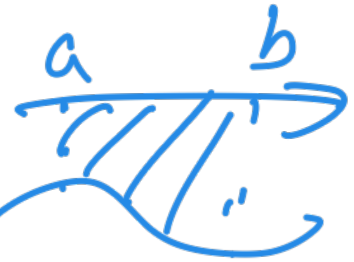
2nd 式を解く(立式)

$$\int_2^3 (x^2-4)\,dx + \int_0^2 -(x^2-4)\,dx$$

$$= \left[\frac{1}{3}x^3 - 4x\right]_2^3 + \left[-\frac{1}{3}x^3 + 4x\right]_0^2$$

$$= \frac{1}{3}(3^3 - 2^3) - 4(3-2) - \frac{1}{3}(2^3) + 4(2)$$

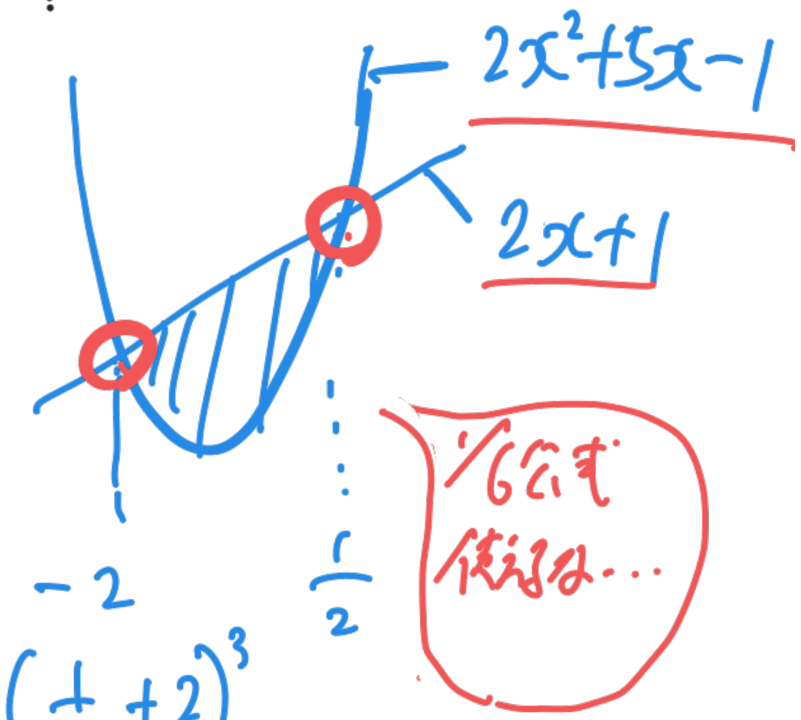
$$= \frac{23}{3}$$



$$\int_a^b f(x)\,dx \qquad \int_a^b -f(x)\,dx$$

問4 次の曲線や直線で囲まれた部分の面積を求めよ

(1) $y = 2x^2 + 5x - 1$, $y = 2x + 1$



$$= 2\left(x + \frac{5}{4}\right)^2 \cdot - \cdot$$

$$2x^2 + 5x - 1 = 2x + 1$$

$$2x^2 + 3x - 2 = 0$$

$$(2x - 1)(x + 2) = 0$$

$$\begin{matrix} 2 & -1 & -1 \\ 1 & 2 & 4 \end{matrix}$$

$$S = \int_{-2}^{\frac{1}{2}} \left\{(2x+1) - (2x^2 + 5x - 1)\right\} dx$$

$$= \int_{-2}^{\frac{1}{2}} \left\{-2x^2 - 3x + 2\right\} dx$$

$$= -2 \int_{-2}^{\frac{1}{2}} (x+2)\left(x - \frac{1}{2}\right) dx$$

$$y = -2 \cdot \left(-\frac{1}{6}\right)\left(\frac{1}{2} + 2\right)^3$$

$$= \frac{1}{3}\left(\frac{5}{2}\right)^3 = \frac{125}{24}$$

(2) $y = x^2 - 3x - 2$.　$y = -x^2 + 2x + 1$

$$\left(\begin{aligned} &x^2 - 3x - 2 = -x^2 + 2x + 1 \\ &2x^2 - 5x - 3 = 0 \\ &(2x+1)(x-3) = 0 \\ &\quad x = -\tfrac{1}{2}, 3 \end{aligned} \qquad \begin{array}{ccc} 2 & 1 & 1 \\ 1 & -3 & -6 \end{array}\right)$$

$$\int_{-\frac{1}{2}}^{3} \{(-x^2+2x+1) - (x^2-3x-2)\}\,dx$$

$$= \int_{-\frac{1}{2}}^{3} (-2x^2+5x+3)\,dx$$

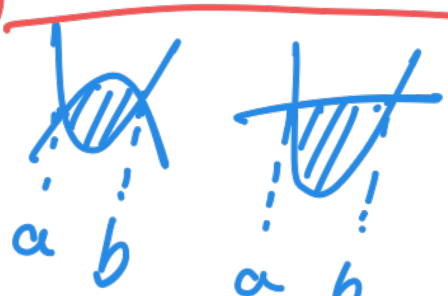
$$= -2\int_{-\frac{1}{2}}^{3} (x-3)\left(x+\frac{1}{2}\right)dx$$

$$= -2\left(-\frac{1}{6}\right)\left\{3-\left(-\frac{1}{2}\right)\right\}^3$$

$$= -2\left(-\frac{1}{6}\right)\left(\frac{7}{2}\right)^3$$

$$= \frac{1}{3}\cdot\frac{343}{8} = \underline{\frac{343}{24}}\ //$$

Point



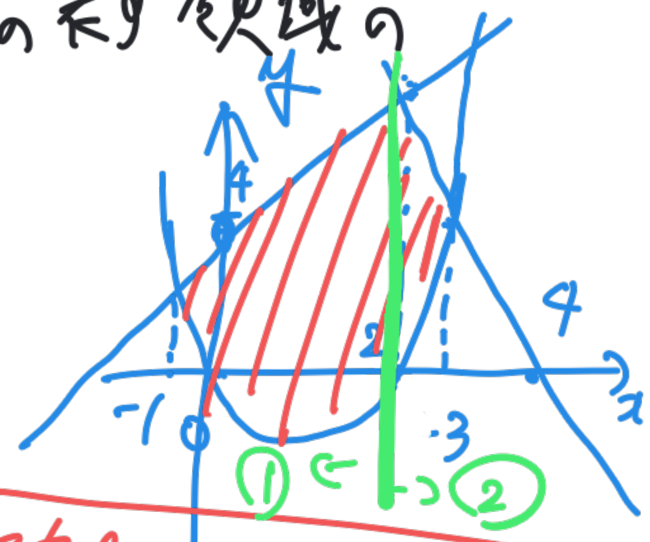
$\Rightarrow$ $1/6$ 公式の利用を考える

但し，最高次係数は外に出して考える．！

問5. 連立不等式 $y \geqq x^2-2x$, $y \leqq x+4$, $y \leqq -3x+12$ の表す領域の面積を求めよ.

① ② ③

① $x(x-2)$



③と①の共有点. $x^2-2x=-3x+12$

$x^2+x-12=0$

$(x+4)(x-3)=0$

$(3,3)$

③と②の共有点.

$x+4=-3x+12$

$4x=8$ $(2,6)$

$x=2$

②と①の共有点.

$x^2-2x=x+4$

$x^2-3x-4=0$

$(x-4)(x+1)=0$

Point いかに正確な位置関係の図が描けるか

① $\displaystyle\int_{-1}^{2}\{(x+4)-(x^2-2x)\}dx$ ② $\displaystyle\int_{2}^{3}\{(-3x+12)-(x^2-2x)\}dx$

$= \displaystyle\int_{-1}^{2}(-x^2+3x+4)dx$ $= \displaystyle\int_{2}^{3}(-x^2-x+12)dx$

$\left[-\frac{1}{3}x^3+\frac{3}{2}x^2+4x\right]_{-1}^{2} + \left[-\frac{1}{3}x^3-\frac{1}{2}x^2+12x\right]_{2}^{3} = \frac{50}{3}$